Test of Convergence

Comparison test

Suppose that $U_n > 0$, $V_n > 0$ and after a finite number of terms

i) $\dfrac{U_n}{U_{n+1}} > \dfrac{V_n}{V_{n+1}}$, then the series $\sum U_n$ is convergent if $\sum V_n$ is Convergent

ii) $\dfrac{U_n}{U_{n+1}} < \dfrac{V_n}{V_{n+1}}$, then the series $\sum U_n$ is divergent if $\sum V_n$ is divergent.

Proof i) $\dfrac{U_n}{U_{n+1}} > \dfrac{V_n}{V_{n+1}}$, where $n \geq m$.

where m is a positive integer.

$$\Rightarrow \frac{U_{n+1}}{U_n} < \frac{V_{n+1}}{V_n}$$

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$$\Rightarrow \frac{u_{n+1}}{v_{n+1}} < \frac{u_n}{v_n}$$

Thus $\frac{u_{n+1}}{v_{n+1}} < \frac{u_n}{v_n} < \ldots < \frac{u_m}{v_m}$

That is $\frac{u_{n+1}}{v_{n+1}} \leq K$ (say),

where K is a finite number

$u_n \leq K v_n$ when $n > m$

Hence if Σv_n is convergent,

Σu_n is also Convergent.

Proof ii.) We have $\frac{u_n}{u_{n+1}} < \frac{v_n}{v_{n+1}}$ where

$n \geq m$, where m is a positive integer

$$\Rightarrow \frac{u_{n+1}}{u_n} > \frac{v_{n+1}}{v_n}$$

$$\Rightarrow \frac{U_{n+1}}{V_{n+1}} > \frac{U_n}{V_n}$$

$$\frac{U_{n+1}}{V_{n+1}} > \frac{U_n}{V_n} > \cdots > \frac{U_m}{V_m}$$

That is $\frac{U_{n+1}}{V_{n+1}} > K$ (Say)

where $K > 0$

$\therefore U_n > K V_n$ where $n > m$

Hence if ΣV_n is divergent

ΣU_n is also divergent.